【Chapter 2 Particle properties of waves】

Electronics: particles ⟶ charge , mass Wave?

Electromagnetic wave: wave ⟶ diffraction, interference,

Polarization particle?

Wave-particle Duality

【2.1Emwaves】

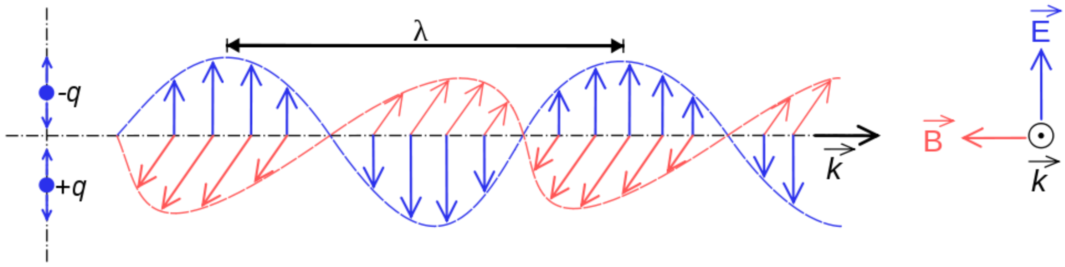

Figure2.1 The electric and magnetic fields in an electromagnetic wave vary together. The fields are perpendicular to each other and to the direction of the wave.

Changing magnetic field ⟶ current (or voltage)

Maxwell proposed: changing electric field ⟶ magnetic field

Hertz created EM waves and determined the wavelength and speed of the wave, and showed that they both have E and B component, and that they could be reflected, refracted, and diffracted ⟶ Wave characteristic.

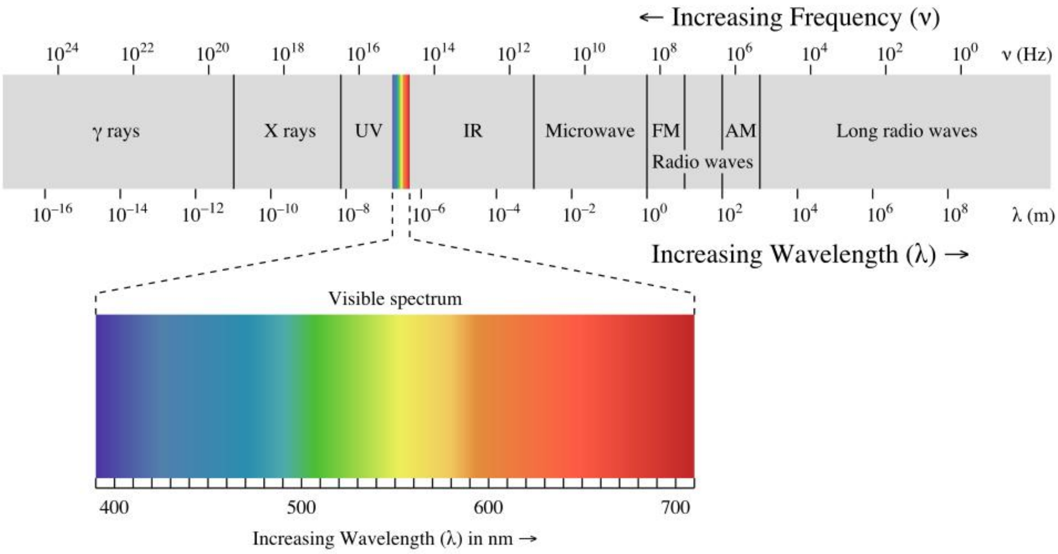

Figure 2.2 The spectrum of electromagnetic radiation.

Principle of Superposition:

When two or more waves of the same nature travel past a point at the same time, the instantaneous amplitude is the sum of the instantaneous amplitude of the individual waves.

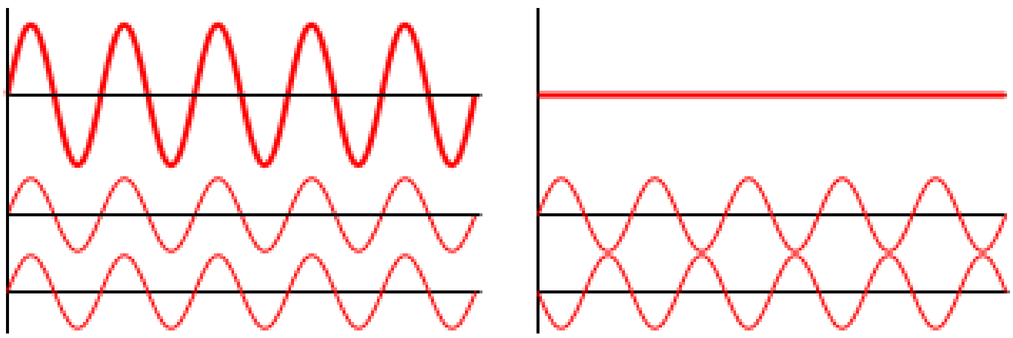

Figure2.3 (a) In constructive interference, superposed waves in phase reinforce each other. (b) In destructive interference, waves out of phase partially or completely cancel each other.

Constructive interference ⟶ same phase, greater amplitude

Destructive interference ⟶ different phase, partial or completely cancellation of waves

Interference ⟶ wave characteristic

Young's diffraction experiments:

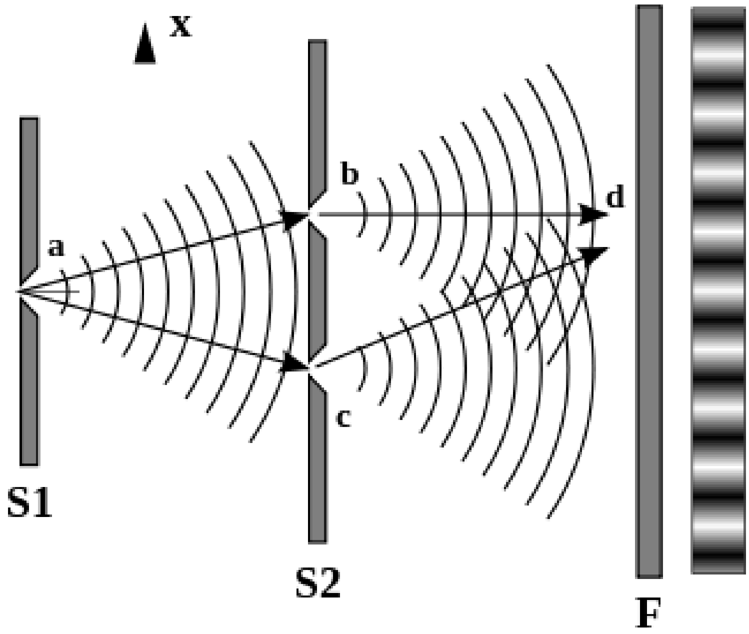

Figure2.4 Origin of the interference pattern in Young's experiment. Constructive inference occurs where the difference in path lengths from the slits to the screen is 0,λ ,2λ ,…Destructive interference occurs where the path difference isλ /2, 3λ /2, 5λ /2…

Diffraction⟶ wave characteristic

【2.2Blackbody radiation 】

Is light only consistent of waves? Amiss: understand the origin of the radiation emitted by bodies of matter.

Blackbody: a body that absorbs all radiation incident upon it, regardless of frequency.

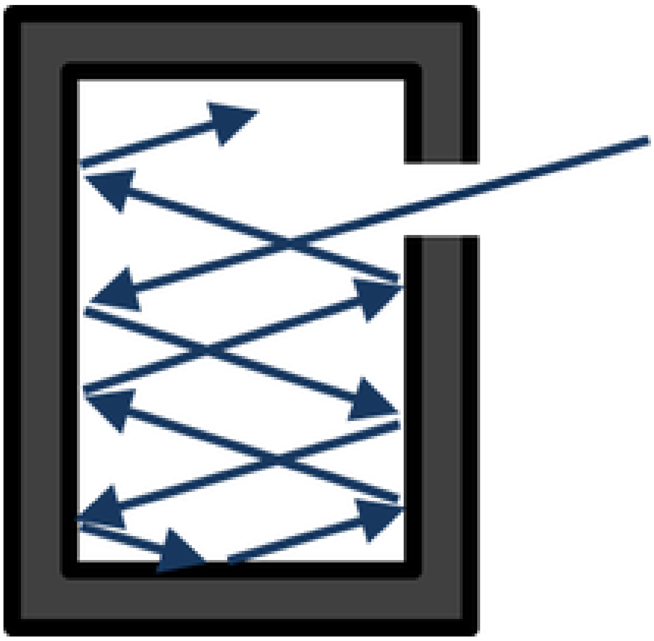

Figure2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

A blackbody radiates more when it is hot than it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than that of a cooler one.

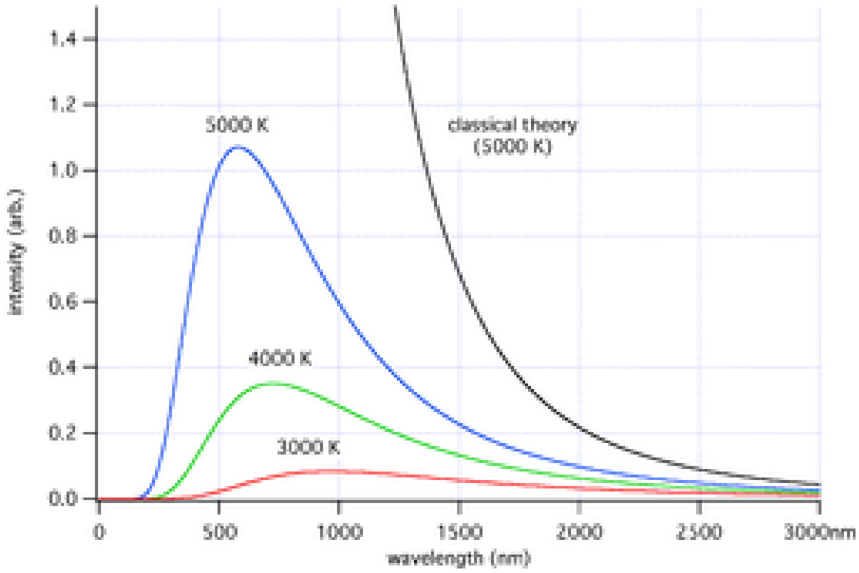

Figure2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body.

Considering the radiation inside a cavity of absolute temperature T whose walls are perfect reflectors to be a series standing EM waves.

$$L = n * \lambda / 2$$

Figure2.7 Em radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L.

Density of standing waves in cavity

$$G(\nu)d\nu = 8\pi\nu^2 d\nu / c^3$$

The higher ν, the shorter the wavelength, and the greater the number of possible standing waves.

The average energy per degree of freedom of an entity that is a member of a system of such entities in thermal equilibrium at T is 1/2kT. K is Boltzmann's constant=1.381*10^{-23}J/k

An idea gas molecular has three degree of freedom: kinetic energy in three independent directions ⟶ 3/2kT

One dimensional harmonic oscillator has two degree of freedom: kinetic energy and potential energy.

Each standing wave in a cavity originates in an oscillating electric charge in the cavity wall. ⟶ Two degree of freedom.

Classic average energy per standing wave $\varepsilon = kT$

Total energy per unit volume in the cavity in ν and ν+dν

$$u(\nu)d\nu = \varepsilon G(\nu)d\nu = (8\pi kT/c^3)\nu^2 d\nu$$

Rayleigh-Jeans formula

ν increase ⟶ energy density increase with ν^2.

In the limit of infinitely high frequencies, u(ν)dν goes to infinity. In reality, the energy density(and the radiation rate)falls to 0 as ν goes to infinity. Ultraviolet catastrophe**Plank**

→

Radiation Formula

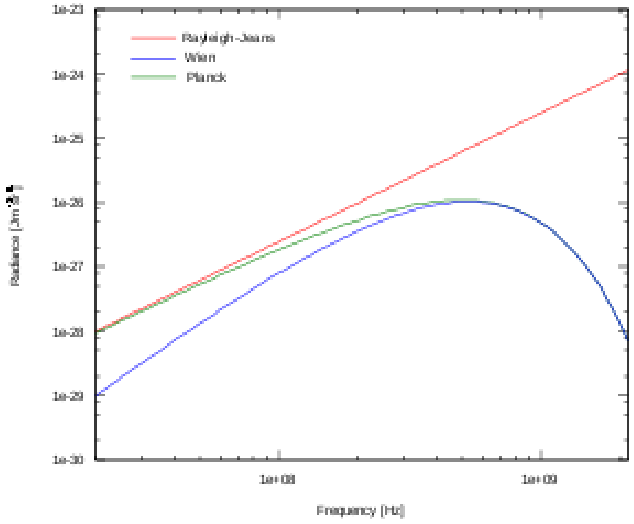

Figure2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum. The discrepancy is known as the ultraviolet catastrophe because it increases with increasing frequency. This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is hν .

$u(\nu)d\nu=(8\pi h/c^3)(\nu^3 d\nu)/(e^{h\nu/kT}-1)$

h is planck's constant=$6.626*10^{-34}$Js

$h\nu >> kT \rightarrow e^{h\nu/kT} \rightarrow \infty \rightarrow u(\nu) \rightarrow 0$

No ultraviolet catastrophe.

In general, $e^x=1+x+x^2/2+$.........

When $h\nu << kT$, $1/(e^{h\nu/kT}-1)\sim 1/((1+(h\nu/kT)-1)\sim kT/h\nu$

$u(\nu)d\nu \sim (8\pi h/c^3)(\nu^3 d\nu)/(kT/h\nu)\sim(8\pi kT/c^3)\nu^2 d\nu$

which is Rayleigh-Jeans formula.

How to justify the Plank radiation formula

The oscillators in the cavity walls could not have a continuous Distribution of possible energy ε but must have only specific energies **$\varepsilon_n = nh\nu$ n=0,1,2……**

An ocillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumpsto the next higher state when it absorbs radiation of ν. Each discrete bundle of energy $h\nu$ is called a quantum.

With oscillator energies limited to $nh\nu$, the average energy per oscillator in the cavity walls turn out to be not kT as for a continuous distribution of oscillator energies, but

$\varepsilon = h\nu/(e^{h\nu/kT}-1)$ average energy per standing wave

【2.3 Photoelectric effect】

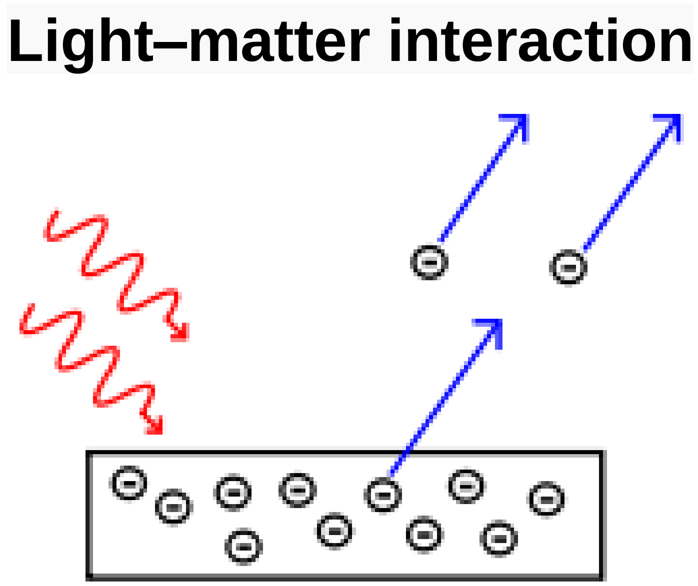

Figure2.9　Experimental observation of the photoelectric effect.

Some of photoelectrons that emerge from the metal surface have enough energy to reach the cathode despite its negative polarity

→ Current

When V is increased to a certain value V_0, no more photoelectrons arrive. V_0 correspond to the max photoelectron kinetic energy.

Three experimental finding:

(1)No delay between the arrival of the light at the metal surface and the emission of photoelectrons.

(2) A bright light yields more photoelectrons than a dim one,

but highest electron energy remain the same.

(3)The higher the frequency of the light, the more energy the photoelectrons have. At the frequencies smaller than ν_0, which is a characteristic of the specific metal, no more electrons are emitted.

Figure2.10 Photoelectron current is proportional to light intensity I for all retarding voltages. The extinction voltage $\vee_0$, which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency$\vee$.

Figure2.11 The extinction voltage$\vee_0$,and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is $\vee=0$, the photoelectron current is the same for light of a given intensity regardless of its frequency.

Quantum theory of light

Einstein proposed **Photons.** The energy in light is not spread out, but is concentrated in small packets. Each photon of light of frequency ν has the energy $h\nu$.

Einstein proposed that energy was not only given to em waves in separate quanta but was also carried by the waves in separate quanta.

Explanation of experiments:

(1)Since em wave energy is concentrated in photons and not spread out, there should be no delay in the emission.

(2)All photons of frequency ν have the same energy $h\nu$. Changing the intensity of light only change the number of photoelectrons but not their energy.

(3)The higher ν, the greater photon energy and so the more energy the photoelectrons have.

ν_o corresponds to the min energy Φ for the electron to escape from the metal surface. This energy is called **work function**.

$$\mathbf{\Phi = h\nu_o}$$

Photoelectric effect $\mathbf{h\nu = kE_{max} + \Phi}$

$$h\nu = kE_{max} + h\nu_o$$

$$kE_{max} = h(\nu - \nu_o).$$

Photo energy

$E=(6.626*10^{-34}Js/1.602*10^{-19}J/eV)\nu=(4.136*10^{-15})\nu eVs$

$\nu=c/\lambda$ $E=1.24*10^{-6}eVm/\lambda$

Example2.2

Ultraviolet light of wavelength 350nm and intensity 1.00W/m^2 is directed at a potassium surface. (a) Find the maximum KE of the photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2?

Solution

(a) From Eq.(2.11) the energy of the photons is, since 1nm=10^{-9}m,

$$E_P=\frac{1.24\times10^{-6}eV\cdot m}{(350nm)\left(\frac{10^{-9}m}{nm}\right)}=3.5eV$$

Table 2.1 gives the work function of potassium as 2.2eV, so

$$KE_{max}=h\nu-\psi=3.5eV-2.2eV=1.3eV$$

(b) The photo energy in joules is 5.68×10^{-19}J. Hence the number of photons that reach the surface per second is

$$n_p=\frac{E/t}{E_p}=\frac{(P/A)(A)}{E_P}=\frac{(1.00w/m^2)(1.00\times10^{-4}m^2)}{5.68\times10^{-19}J/photon}$$

$$=1.76\times10^{14}photons/s$$

The rate at which photoelectrons are emitted is therefore

$$n_e=(0.0050)n_p=8.8\times10^{11}photoelectrons/s$$

- **What is light**

Wave model: light intensity $\propto \overline{E^2}$

Particle model: light intensity $\propto$ N(#of photons/sec.area)

$$N \propto \overline{E^2}$$

.N is large ⟶ interference pattern

N is small ⟶ a series of random flashes

.if keep track of flashes for long time

⟶ same as large N

⟶ intensity of wave at a given place on the specific space $\propto$ the probability of finding photons.

Wave & quantum theory complement each other.

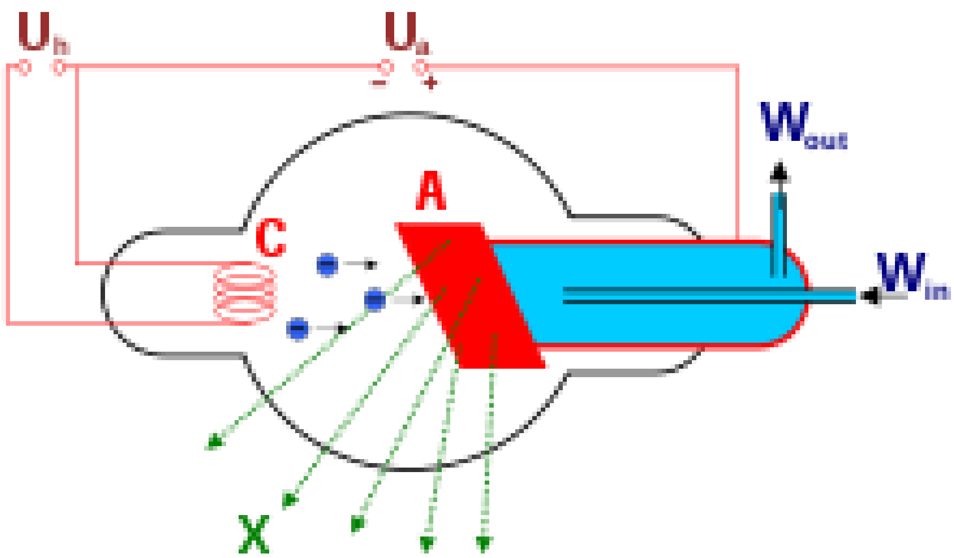

Figure2.15 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.

photoelectric effect : $E_{photons} \longrightarrow E_{e'}$

yes $\longrightarrow$ x-ray

faster e′ $\longrightarrow$ more x-ray

$\longrightarrow$

# of e′ increase $\quad$ Intensity of x-ray increase

.for given accelerating V $\longrightarrow \lambda_{min}$

V $\uparrow \longrightarrow \lambda_{min} \downarrow$

.most of e′ $\longrightarrow$ heat

A few e′ lose E in single collisions $\longrightarrow$ x-ray

Figure2.16 X-ray spectra of tungsten at various accelerating potentials.

- .x-ray are em waves

EM theory predicts that an accelerated electric charge will radiate em waves, and a rapidly moving e′ suddenly brought to rest is certainly accelerated $\longrightarrow$ Bremsstranlung (“braking radiation”)

Figure2.17 X-ray spectra of tungsten and molybdenum at 35kV accelerating potentials.

x-ray at specific $\lambda \longrightarrow$ nonclassical

* different targets give different characteristic x-ray

* for the same V, λ_{min} is the same for different materials

$$\lambda_{min}=(1.24\times10^{-6})/V(m)$$

$h\nu_{max} = Ve = hc/\lambda_{min} \longrightarrow \lambda_{min} = hc/Ve = (1.24\times10^{-6})/V$

Figure2.18　The scattering of electromagnetic radiation by a group of atoms. Incident plane waves are reemitted as spherical waves.

Scattering by an atom (wave model)

atom in E $\longrightarrow$ polarized $\longrightarrow$ distorted charge distribution

$\longrightarrow$ electric dipole

em wave with ν on atom $\longrightarrow$ polarization charge with ν

$\longrightarrow$ oscillating electric dipole $\longrightarrow$ radiate em wave

Figure2.19　Two sets of Bragg planes in an NaCl crystal.

x-ray falls on a crystal will be scattered in all directions because of regular arrangement of atoms $\longrightarrow$ constructively interference

$\longrightarrow$ Bragg’s condition ($2d\sin\theta=\lambda$)

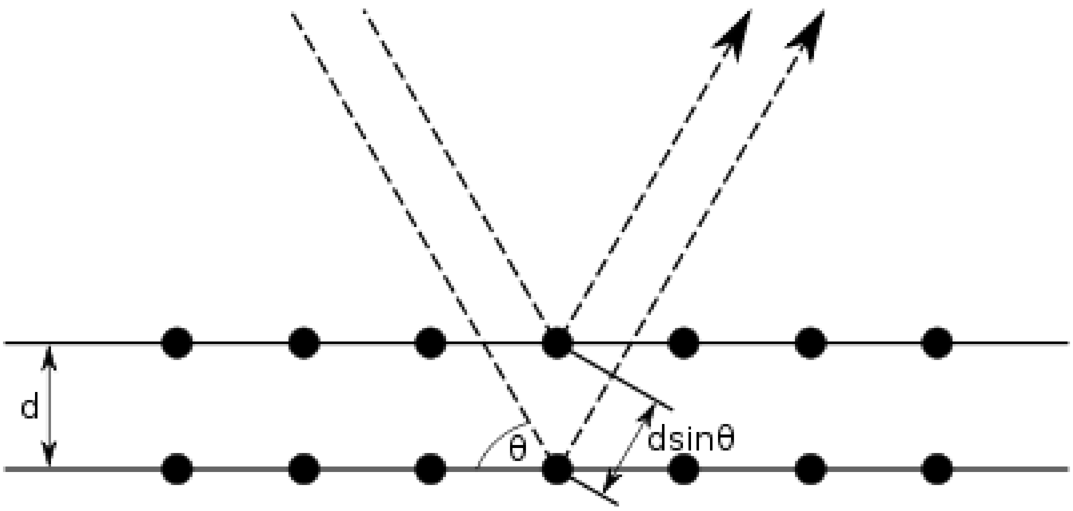

Figure2.20 X-ray scattering from a cubic crystal.

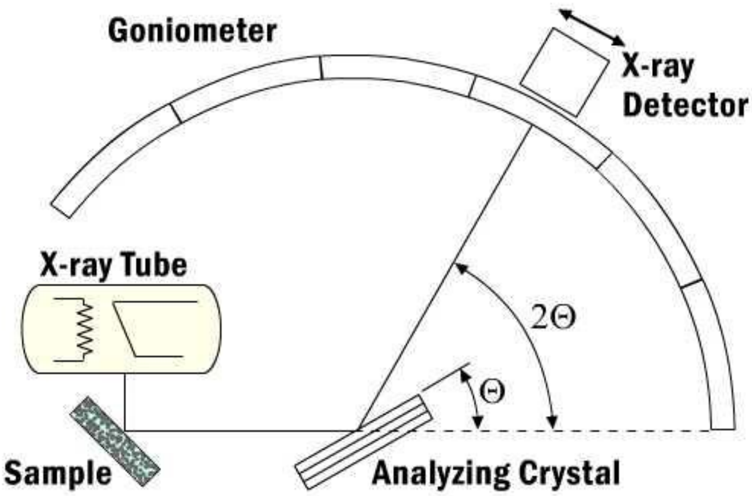

Figure2.21 X-ray spectrometer.

- **Compton effect**

Loss in photon energy=gain in e´ energy

$$h\nu - h\nu' = kE$$

for massless particle E= Pc (P=momentum)

⟶ photon momentum P =E/c = hν/c

Figure2.22 (a) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (b) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Figure2.23 Experimental demonstration of the Compton effect.

$h\nu/c = (h\nu'/c)\cos\Phi + P\cos\theta$ (parallel)……..(1)

$0 = (h\nu'/c)\sin\Phi - P\sin\theta$ () ……………….(2)

(1)&(2)x c

$Pc(\cos\theta) = h\nu - h\nu'\cos\Phi$

$Pc(\sin\theta) = (h\nu')\sin\Phi$

$P^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\Phi + (h\nu')^2$

& $E = KE + m_oc^2$ $E = \sqrt{m_o^2c^4 + P^2c^2}$

$\longrightarrow (KE + m_oc^2)^2 = m_o^2c^4 + P^2c^2$

$P^2c^2 = KE^2 + 2m_oc^2KE$

Because $KE = h\nu - h\nu'$

$\longrightarrow P^2c^2 = (h\nu - h\nu')^2 + 2m_oc^2KE$

$2m_oc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos\Phi)$……(3)

(3)/$2h^2c^2$ $m_oc/h(\nu/c - \nu'/c) = (\nu/c)(\nu'/c)(1 - \cos\Phi)$

$\longrightarrow m_oc/h(1/\lambda - 1/\lambda') = (1 - \cos\Phi)/(\lambda\lambda')$

$\longrightarrow \lambda' - \lambda = (h/m_oc)(1 - \cos\Phi) = \lambda c(1 - \cos\Phi)$

λ=Compton wavelength

- **Relativistic formulas**

Total energy $E=\frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ m_o= rest mass

Relativistic momentum $P = \frac{m_o v}{\sqrt{1-v^2/c^2}}$

When $m_o = 0$ (massless particle) & v<c ⟶ E=P=0

How about v=c , m_o= 0 ⟶ E=0/0 , P=0/0 (any values)

$E^2=\frac{m_o^{\,2}c^4}{1-v^2/c^2}$, $P^2=\frac{m_o^{\,2}v^2}{1-\frac{v^2}{c^2}}$ ⟶ $P^2c^2 = \frac{m_o^{\,2}v^2}{1-\frac{v^2}{c^2}}c^2$

⟶ $E^2 - P^2c^2 = \frac{m_o^{\,2}c^4}{1-v^2/c^2}(1-v^2/c^2) = m_o^{\,2}c^4$

⟶ $E^2 = m_o^{\,2}c^4 + P^2c^2$

⟶ **For all particles** **E=**$\sqrt{m_0^{\,2}c^4 + p^2c^2}$ **=** $\sqrt{E_o^{\,2} + p^2c^2}$

Restriction of massless particles : E = Pc (m_o=0)

Total energy $mc^2=m_oc^2 + KE = \frac{m_o c^2}{\sqrt{1-v^2/c^2}}$

Figure2.24 Compton scattering.

- **Pair production**

Figure2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

A photon give an e′ all of its energy ⟶ photoelectric

part of its energy ⟶ compton

a photon materialize into an e′ & position

(momentum is conserved with the help of the nucleus which carries away enough photon momentum)

Figure2.26 Vector diagram of the momenta involved if a photon were to materialize into an electron-positron pair in empty space. Because such an event cannot conserve both energy and momentum, it does not occur. Pair production always involves an atomic nucleus that carries away part of the photon momentum.

rest energy m_oc^2 of electron or position is 0.51Mev ⟶ pair production requires a photon energy ≥ 1.02Mev

- **pair production cannot occur in empty space**

concervation of energy $h\nu = 2mc^2$

momentum concervation $h\nu/c = 2P\cos\theta$ ⟶ $h\nu = 2Pc(\cos\theta)$

$P = mv$ ⟶ $h\nu = 2mc^2(v/c)\cos\theta$

$$v/c < 1 \ \& \ \cos\theta \leq 1 \longrightarrow h\nu < 2mc^2$$

Figure2.27　X- and gamma rays interact with matter chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.

Figure2.28　The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

linear attenuation coefficient

$$\frac{-dI}{I} = udx \longrightarrow I = I_o \exp(-ux)$$

absolute thickness $$x = \frac{\ln(I_o/I)}{u}$$

Figure2.29　Linear attenuation coefficients for photons in lead.